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Dynamical Symmetry Breaking with Large Anomalous Dimension in Gauge Theories

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Abstract

An analysis is given of the dynamical symmetry breaking of semi-simple gauge groups. We construct a class of renormalizable gauge theories for the dynamically broken topcolor and technicolor interactions. It is shown that a four-Fermi interaction in the strong coupling phase emerges by the tumbling of semi-simple gauge groups in the low energy region. In our models the topcolor interaction provides the top quark with a large anomalous dimension.

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1 Introduction

The dynamical symmetry breaking scenario of the Standard Model is a fascinating issue. Accordingly, the technicolor models[1] and the top quark condensation models[2] are considered. However, there are theoretical and experimental difficulties in many models.

The simplest technicolor models are excluded by the challenges of the oblique corrections in the W, Z gauge boson self-energies,[3] and so are even for the walking technicolor models.[4] Then, the candidates for an acceptable technicolor model will have spontaneously broken dynamics or have the techni-fermions with the standard gauge symmetry invariant mass[5, 6]. However, in turn, we must trade naturalness for the vanishing oblique corrections. The flavor changing neutral current processes are also a problem[7] to be overcome when we explain the masses of the ordinary fermions by sideways mechanism[8]. We encounter the light pseudo Nambu-Goldstone (NG) bosons when we use more than one doublet of techni-fermions.

The top condensation model[2] has severe problems of naturalness and renormalizability, although the model can satisfy all phenomenological constraints so far. The phenomenological success is due to the dynamics providing a large anomalous dimension $\gamma_m = 2$ to the top quark bilinear operator $\bar{t}t$.[9] When we formulate the model as a renormalizable gauge theory without scalars, we are forced to introduce a strong coupling interaction, such as technicolor, which dynamically breaks the topcolor gauge symmetry.

Recently, a technicolor model assisted by the topcolor model was proposed[10] in order to explain the large top quark mass and the naturalness of the broken topcolor interactions. In such a model the technicolor interactions are responsible for the masses of the W and Z gauge bosons as well as the top-gluon. The top quark mass is dynamically generated by the top quark condensation and the masses of the other fermions are provided by extended technicolor sideways. However, many problems still remain unsolved.[11]

In this paper, we show how to construct a class of topcolor assisted technicolor models in the framework of the Schwinger-Dyson equation in the improved ladder approximation. We can also construct a renormalizable top quark condensation model. Our theoretical models have the following properties; the renormalizability, large top quark mass, the large anomalous dimension $\gamma_m \simeq 2$.

The top quark condensation model is based on the works in Ref. [12]. It is shown that asymptotically free gauge theories with an additional four-Fermi interaction has a non-trivial ultraviolet fixed point and the large anomalous dimension within the (improved)

ladder approximation. The present work is an extension of that work in part. Our work is essentially based on that in Ref. [13]. Semi-simple gauge groups are used for the tumbling gauge theory. One gauge symmetry, which is a simple subgroup of the gauge group, is broken by the gauge interaction of the other gauge symmetry. We find the complete phase structure of the tumbling gauge theories with semi-simple unitary gauge group.

This paper is organized as follows. In section 2 we study the dynamical symmetry breaking of the semi-simple unitary group $SU(N_A) \times SU(N_B)$ in the framework of the Schwinger-Dyson equation, and find the phase structure. A system appears with an asymptotically free gauge interaction and a four-Fermi interaction. The detailed form of the coupled Schwinger-Dyson equation is given in section 3. In section 4 we briefly show how the Nambu-Goldstone bosons couple to the gauge currents. The decay constants are given in terms of the fermion mass functions. In section 5 we solve a Schwinger-Dyson equation for the top quark in the improved ladder approximation and show that the top quark four-Fermi interaction appears in the strong coupling phase.

2 Dynamical Symmetry Breaking in Gauge Theories

Although intuitive pictures [14, 15] of dynamical gauge symmetry breaking are already given, there is an unsolved problem especially in semi-simple gauge group[13]. What is the phase structure of such a system? In this section we study the dynamical breaking of a semi-simple gauge symmetry and solve the problem.

To begin with, we consider the semi-simple unitary gauge group $G = SU(N_A)_A \times SU(N_B)_B$ for simplicity. The gauge bosons of $SU(N_A)_A$ and $SU(N_B)_B$ are denoted by A^a_μ and B^a_μ, respectively. It will be also interesting in general to consider anomaly safe groups having complex representations such as $SO(10)$, $SO(14)$, $\cdots$ and E_6. We introduce three kinds of fermions ψ_R, ξ_L and η_L transforming as $\psi_R \sim (\underline{N}_A, \underline{N}_B)$, $\xi^i_L \sim (\underline{N}_A, 1)$ and $\eta^j_L \sim (1, \underline{N}_B)$ for each i $(= 1, \cdots, N_B)$ and j $(= 1, \cdots, N_A)$ where $\underline{N}$ represents the fundamental representation of the unitary group $SU(N)$. (see also Table. 1.) The fermions ξ^i_L are $SU(N_B)_B$ singlets and the fermions η^j_L are $SU(N_A)_A$ singlets. The subscripts R and L denote the usual chiral projections. The gauge symmetry G has no anomaly with this choice of matter fields. Then, the system consists of two gauge bosons and the three types of fermions which minimally couple to the gauge bosons according to their representations. There is a global symmetry $SU(N_B)_\xi \times SU(N_A)_\eta$ acting on these ξ^i_L and η^j_L, since fermions ξ^1_L, $\cdots$, $\xi^{N_A}_L$ are massless N_B $\underline{N}_A$-plets and η^1_L, $\cdots$, $\eta^{N_B}_L$ are massless

N_A $\underline{N}_B$-plets under $SU(N_A)_A$ and $SU(N_B)_B$, respectively. We may regard this global symmetry as a weak gauge symmetry by adding the corresponding gauge bosons, which is irrelevant in the present consideration of dynamical symmetry breaking. The charge assignments of the fermions are summarized in Table 1.

	$SU(N_A)_A$	$SU(N_B)_B$	$SU(N_B)_\xi$	$SU(N_A)_\eta$
ψ_R	N_A	N_B	1	1
ξ_L	N_A	1	N_B	1
η_L	1	N_B	1	N_A

Table 1: The charge assignments of the fermions.

We first consider the extreme case where the $SU(N_B)_B$ gauge coupling is turned off and only the $SU(N_A)_A$ gauge symmetry is relevant. We have a condensate $\langle \overline{\xi}_L \psi_R \rangle$ driven by the $SU(N_A)_A$ gauge interaction. The most attractive channel is obvious in analogy with QCD. The condensate $\langle \overline{\xi}_L \psi_R \rangle$ implies that the N_B pairs of two Weyl fermions ξ_L^i and ψ_R^i combine to form the N_B massive Dirac fermions as

$$\Psi_A^i \equiv \begin{pmatrix} \psi_R^i \\ \xi_L^i \end{pmatrix} , \tag{2.1}$$

where the superscript of ψ_R^i is the index of the gauge group $SU(N_B)_B$. Owing to the custodial symmetry $SU(N_B)_B \times SU(N_B)_\xi$, the condensate takes the form $\langle \overline{\Psi}_{Ai} \Psi_A^j \rangle \propto \delta_i^j$ without loss of generality. This condensate breaks the symmetry $SU(N_B)_B$ completely, or more precisely, breaks $SU(N_B)_B \times SU(N_B)_\xi$ down to the diagonal subgroup $SU(N_B)_{B+\xi}$. Accordingly the NG boson π_A^a of the $SU(N_B)_{B+\xi}$ adjoint representation appears, and the $SU(N_B)_B$ gauge boson becomes a massive vector field of $SU(N_B)_{B+\xi}$ adjoint representation. The same arguments hold for the opposite case where only the $SU(N_B)_B$ gauge coupling is switched on. In turn, the condensate $\langle \overline{\eta}_L \psi_R \rangle$ leads to the Dirac fermions

$$\Psi_B^j \equiv \begin{pmatrix} \psi_R^j \\ \eta_L^j \end{pmatrix} , \tag{2.2}$$

where the superscript of ψ_R^j is the index of the gauge group $SU(N_A)_A$. The condensate breaks the symmetry $SU(N_A)_A \times SU(N_A)_\eta$ down to the diagonal subgroup $SU(N_A)_{A+\eta}$, and the NG boson π_B^a of the $SU(N_A)_{A+\eta}$ adjoint representation appears.

Now, let us consider the generic case in which both gauge couplings of G are turned on. Although the physical picture is rather transparent[14, 15, 13] in analogy with the chiral symmetry breaking of QCD, the detailed feature of dynamically breaking the gauge symmetry is complicated. We have a possibility that both the gauge symmetries of $SU(N_A)_A$ and $SU(N_B)_B$ are dynamically broken by the condensates $\langle\overline{\eta}_L\psi_R\rangle$ and $\langle\overline{\xi}_L\psi_R\rangle$. The resultant manifest symmetry is the global symmetry $SU(N_A)_{A+\eta} \times SU(N_B)_{B+\xi}$. This global symmetry is vector-like and cannot be broken because of the Vafa-Witten theorem[16]. As will be shown, the dynamical $SU(N_A)_A$ symmetry breaking is solely caused by the (broken) $SU(N_B)_B$ gauge interaction, and simultaneously the dynamical $SU(N_B)_B$ symmetry breaking is solely caused by the (broken) $SU(N_A)_A$ gauge interaction. Accompanied by the dynamical $SU(N_B)_B$ symmetry breaking, the NG boson π_A^a as well as the Dirac fermions Ψ_A and $\overline{\Psi}_A$ are formed by the $SU(N_A)_A$ gauge interaction. This NG boson π_A^a has a derivative coupling to the broken $SU(N_B)_B$ current $J_B^{a\mu}$ with dimensionful coupling strength f_A. This quantity f_A is the decay constant of π_A^a. Owing to the minimal coupling, $g_B J_B^{a\mu} B_\mu^a$, the NG boson π_A^a is absorbed into the $SU(N_B)_B$ gauge boson B_μ^a and this B_μ^a becomes massive vector field of $SU(N_B)_{B+\eta}$ adjoint representation. Here, we write the coupling constants of the $SU(N_A)_A$ and $SU(N_B)_B$ gauge interactions as g_A and g_B, respectively. Simultaneously, the same argument holds for π_B^a, and the $SU(N_A)_A$ gauge boson A_μ^a becomes massive. We write the masses of A_μ^a and B_μ^a as M_A and M_B, respectively. The masses, M_A and M_B, are proportional to the decay constants, f_B and f_A, of the NG bosons, π_B and π_A,[17]:

$$
\begin{aligned}
M_A^2 &= g_A^2 f_B^2 \,, \\
M_B^2 &= g_B^2 f_A^2 \,,
\end{aligned}
\tag{2.3}
$$

respectively in the leading order of couplings.

On the other hand, the decay constants f_A and f_B depends on the gauge boson masses M_A and M_B, since the massive gauge bosons A_μ^a and B_μ^a are responsible for forming the bound states π_A^a and π_B^a. Namely, the gauge boson masses and the decay constants of the NG bosons are consistently determined with each other. It seems very complicated to study the dynamical symmetry breaking systematically and quantitatively with the help of Schwinger-Dyson and Bethe-Salpeter equations. How do we disentangle the relation that output quantities f_A and f_B are also input quantities M_A and M_B?

Our key prescription for this problem is very simple. We tentatively regard the gauge boson masses and the NG boson decay constants as independent. For given masses M_A

and M_B we calculate the decay constant f_A and f_B using the SD and BS equations. We vary the values of M_A and M_B as inputs. Among the resulting sets $\{(M_A, M_B; f_A, f_B)\}$, we search for the desired solution satisfying the relation (2.3). We will explain a more systematic method later.

Moreover, there are one further observation which makes the analysis simpler. As mentioned before, the condensates $\langle\overline{\xi}_L\psi_R\rangle$ and $\langle\overline{\eta}_L\psi_R\rangle$ are *solely* driven by the gauge interactions $SU(N_A)_A$ and $SU(N_B)_B$, respectively. For example, let us consider the propagator $\langle\Psi_A\overline{\Psi}_A\rangle$. The Weyl fermion ξ_L is $SU(N_B)_B$ singlet, and the $SU(N_B)_B$ gauge boson B^a_μ does not interact with the ξ_L component of the Dirac fermion Ψ_A. Then, the gauge boson B^a_μ cannot drive the fermions ξ_L and ψ_R to make the chiral transitions $\xi_L \to \psi_R$ or $\psi_R \to \xi_L$. The chiral transitions $\xi_L \leftrightarrow \psi_R$ are properly driven by the A^a_μ massive gauge boson. The leading order terms of the Schwinger-Dyson equation for $\langle\Psi_A\overline{\Psi}_A\rangle$ consist of the two diagrams with only the A^a_μ massive gauge boson as in Fig. 1. The similar

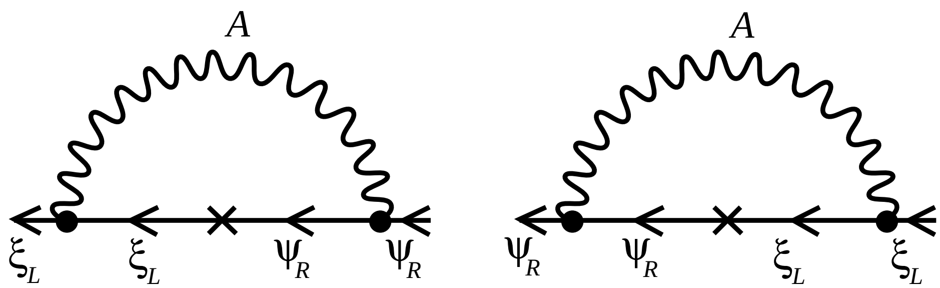

Figure 1: The leading Feynman diagrams for the SD equation for Ψ_A propagator. Only the $SU(N_A)_A$ gauge boson A^a_μ contributes.

argument holds for the propagator $\langle\Psi_B\overline{\Psi}_B\rangle$ where the main contributions for the chiral transitions $\eta_L \leftrightarrow \psi_R$ are given by B^a_μ. More importantly the $\langle\xi_L\overline{\psi}_R\rangle$ propagator receives mixing effects by the condensates $\langle\overline{\eta}_L\psi_R\rangle$ and $\langle\overline{\psi}_R\eta_L\rangle$. The leading effect is depicted in Fig. 2. We take account of all such effects in the coupled Schwinger-Dyson equations.

Let us study the phase diagram of the present system. The coupled Schwinger-Dyson equations are easily solved by using a numerical iteration (relaxation) method. The detailed form will be given in section 3. The initial functional forms for the mass functions are taken as symmetric; i.e., $\Sigma_A(x) = \Sigma_B(x)$. When we evaluate the decay constants, we use a generalized Pagels-Stokar formula which will be derived in section 4. The decay constants f_A and f_B are functions of the gauge boson masses (and the interaction scales); $f_A = f_A(M_A, M_B)$, $f_B = f_B(M_B, M_A)$. Substituting this equations into Eqs. (2.3), we

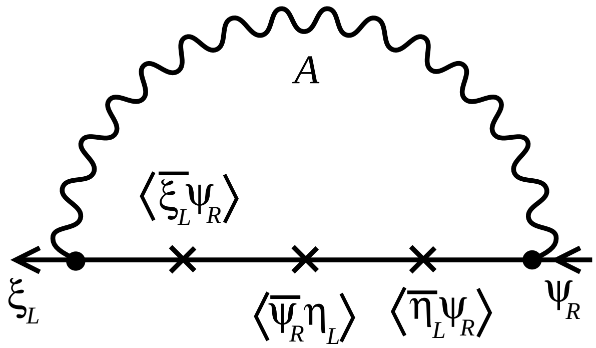

Figure 2: The chiral transition $\langle \xi_L \overline{\psi}_R \rangle$ receives a mixing effect by the condensates $\langle \overline{\eta}_L \psi_R \rangle$ and $\langle \overline{\psi}_R \eta_L \rangle$.

find that the gauge boson masses are determined by the intersection of the following two equations

$$M_A = g_A f_B(M_B, M_A) , \tag{2.4}$$

$$M_B = g_B f_A(M_A, M_B) . \tag{2.5}$$

We can easily calculate the gauge boson masses numerically by applying an iteration method to Eqs. (2.4) and (2.5). In order to make the analysis simple, we neglect the couplings appearing in Eqs. (2.4) and (2.5). The values of M_A and M_B converge fast well. The result is shown in Fig. 3 with $N_A = N_B = 3$. We use a unit scale setting $\Lambda_B = 1$ and fix the value of N_B as $N_B = 3$ below. We observe three vacua at the point $\Lambda_A = \Lambda_B$. It is seen, however, that the symmetric vacuum ($\Sigma_A = \Sigma_B$) is unstable against the perturbation of the couplings. If the symmetric vacuum was one of the stable points of the system, we would have a plateau extending from the point $\Lambda_A = \Lambda_B$ to $\ln(\Lambda_A/\Lambda_B) > 0$ in Fig. 3. We conclude that the symmetric vacuum is an artifact generated by our procedure and is not true vacuum. Then, the correct solution shows a simple first order phase transition at $\ln(\Lambda_A/\Lambda_B) = 0$. Here, we notice that both the $SU(N_A)_A$ and $SU(N_B)_B$ broken vacua are stable against any values of $\ln(\Lambda_A/\Lambda_B)$. We recognize this fact by the explicit forms of the coupled Schwinger-Dyson equations (in Eqs. (3.17)). For example, if we use asymmetric initial functions ($\Sigma_A(x) \neq 0$ and $\Sigma_B(x) \equiv 0$), we will always find the $SU(N_B)_B$ broken vacuum having no dependence of the values of the couplings.

As a result, we have the following phase. In the range $-\infty < \ln(\Lambda_A/\Lambda_B) \leq 0$, the $SU(N_A)_A$ symmetry is broken with the gauge boson mass $M_A = f_B(0, M_A)$ and the symmetry $SU(N_B)_B$ is completely manifest. The values of the masses drastically change

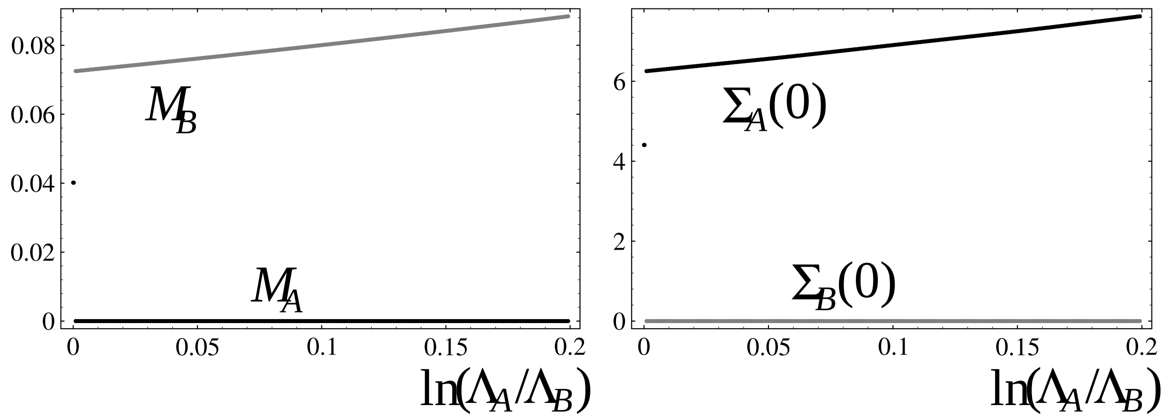

Figure 3: The left hand side is the estimated gauge boson masses and the right hand side is the mass functions $\Sigma_A(0)$ and $\Sigma_B(0)$ in the case $N_A = N_B = 3$. The horizontal axes are the relative strength of the couplings $\ln(\Lambda_A/\Lambda_B)$. The black points indicate M_A, $\Sigma_A(0)$ and the gray points indicate M_B, $\Sigma_B(0)$. A simple first order phase transition occurs at $\ln(\Lambda_A/\Lambda_B) = 0$.

around the point $\ln(\Lambda_A/\Lambda_B) = 0$. In the range $0 \leq \ln(\Lambda_A/\Lambda_B) < \infty$, $SU(N_A)_A$ is completely manifest and $SU(N_B)_B$ is broken with $M_B = f_A(0, M_B)$. This result shows that the most attractive channel (MAC) hypothesis works completely. The broken $SU(N_B)_B$ gauge interaction can form $SU(N_A)_A$ singlet four-Fermi interaction by introducing $SU(N_A)_A$ singlet fermions. (Notice that ψ_R and ξ_L cannot form such a four-Fermi interaction.)

In conclusion, we obtain a theory in which $SU(N_A)_A$ *Yang-Mills and* $SU(N_B)_B$ *four-Fermi interactions appear in the low energy region* by tuning the couplings such that $\ln(\Lambda_A/\Lambda_B) > 0$. There is no fine-tuning. In certain appropriate regions the four-Fermi interaction is in the strong coupling phase necessary for a dynamical symmetry breaking to take place. We will study this in section 5.

Next, let us take different gauge groups; i.e, $N_A \neq N_B$. We fix the $SU(N_B)_B$ gauge group by setting $N_B = 3$. We change the value of N_A as $N_A = 5, 8, 9, 10, 12$. As for the $N_A = 5, 8$ cases we have first order phase transitions. The phase transition points move to the region $\ln(\Lambda_A/\Lambda_B) > 0$ as in Figs. 4 and 5.

However, for the $N_A = 9$ case we have a second order phase transition as in Fig. 6 at $\ln(\Lambda_A/\Lambda_B) \cong 0.1515$. There are two phase transitions at $\ln(\Lambda_A/\Lambda_B) \cong 0.1515$ and 0.152, and the latter one seems to be of first order. Second order phase transitions occur clearly in the $N_A = 10, 12$ cases as in Figs. 7 and 8. These models provide asymptotically free gauge theories with additional strong coupling four-Fermi interaction around a second

order phase transition point. This is studied in Ref. [12].

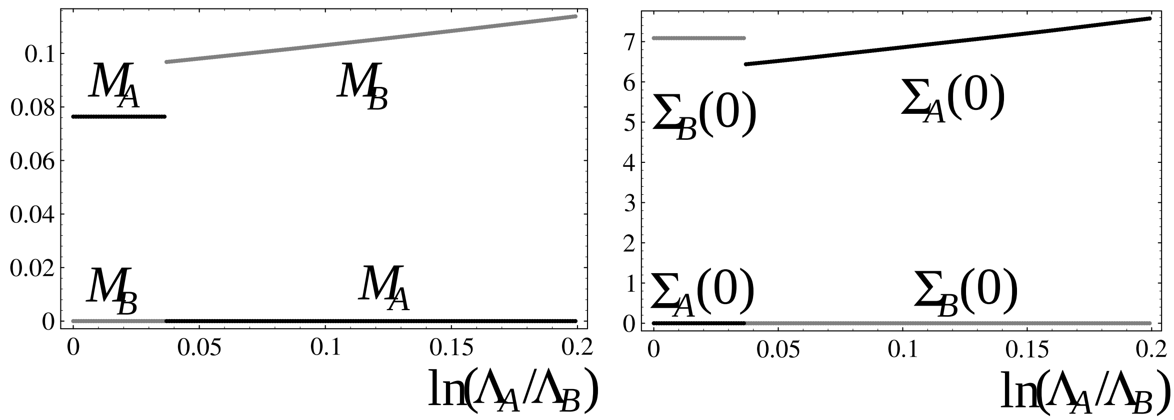

Figure 4: The phase diagram in the case $N_A = 5$, $N_B = 3$. A first order phase transition occurs at $\ln(\Lambda_A/\Lambda_B) = 0.0355$.

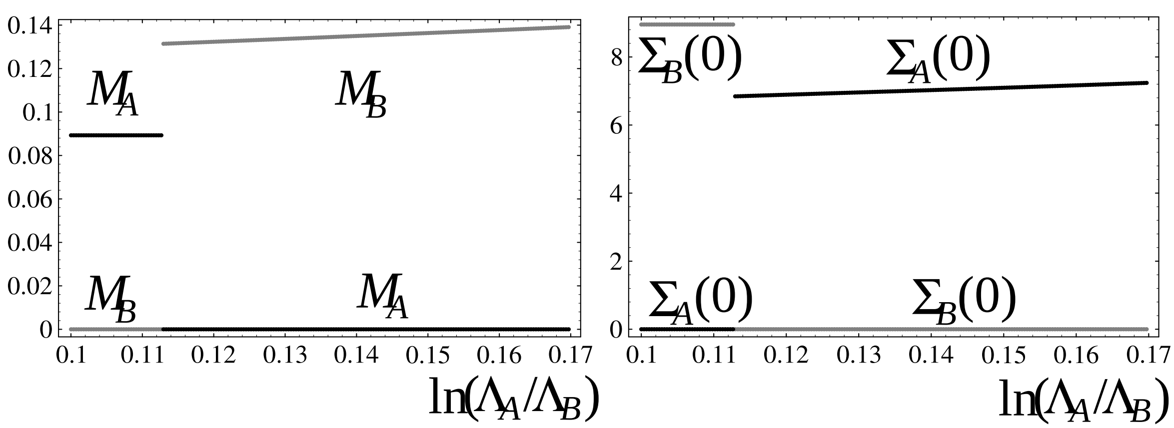

Figure 5: The phase diagram in the case $N_A = 8$, $N_B = 3$. A first order phase transition occurs at $\ln(\Lambda_A/\Lambda_B) = 0.111$.

Here we note a fact. If we use a initial condition such that $\Sigma_A(x) \equiv 0$ and $\Sigma_B(x) \neq 0$, we always have a vacuum where the $SU(N_A)_A$ symmetry is broken and the $SU(N_B)_B$ symmetry is manifest. Similar argument holds for the initial condition such that $\Sigma_A(x) \equiv 0$ and $\Sigma_B(x) \neq 0$. Namely, we can always have two different solutions specified by $\Sigma_A(x) \equiv 0$ and $\Sigma_B(x) \equiv 0$, depending on the initial functional forms of the mass function in solving the coupled Schwinger-Dyson equations. This property gives hysteresis curves in phase diagrams if we use particular forms for the initial mass functions.

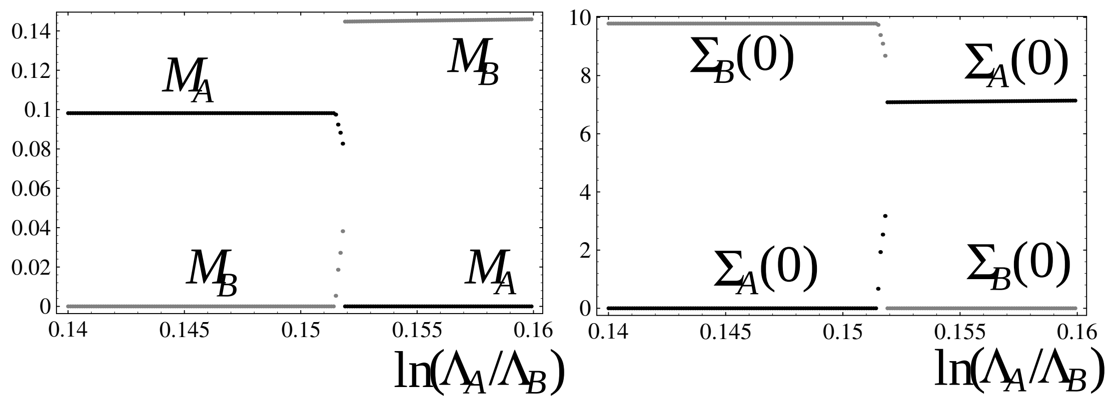

Figure 6: The phase diagram in the case $N_A = 9$, $N_B = 3$. Phase transitions occur around $\ln(\Lambda_A/\Lambda_B) = 0.151 \sim 0.152$. The former one is of second order and the latter one seems to be of first order.

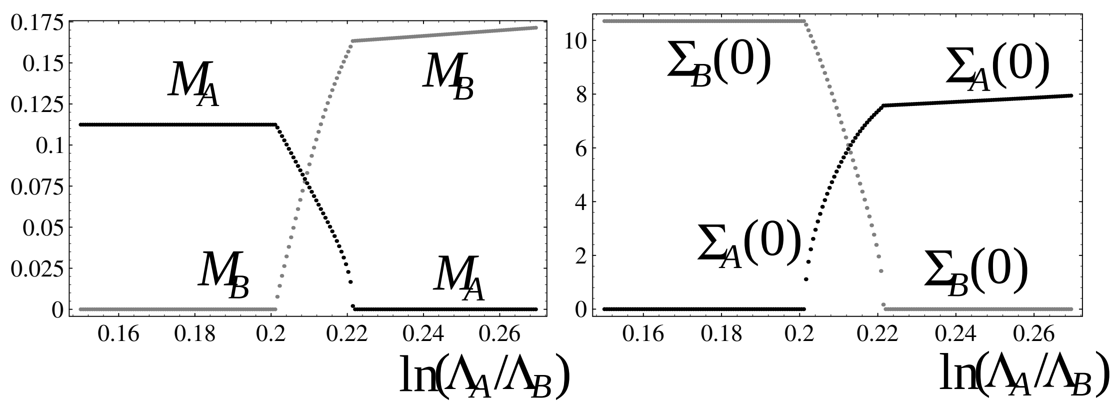

Figure 7: The phase diagram in the case $N_A = 10$, $N_B = 3$. Second order phase transitions occur at $\ln(\Lambda_A/\Lambda_B) = 0.201, 0.222$.

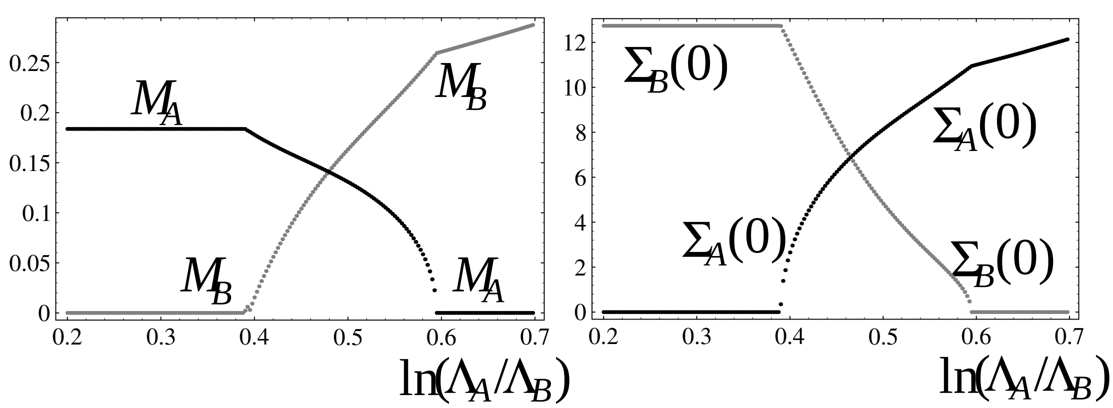

Figure 8: The phase diagram in the case $N_A = 12$, $N_B = 3$. Second order phase transitions occur at $\ln(\Lambda_A/\Lambda_B) = 0.387, 0.597$.

3 The Coupled Schwinger-Dyson Equations

In this section, we derive the coupled Schwinger-Dyson equations used in the above analysis. Before proceeding, we explain the basic ingredients in the Schwinger-Dyson equations here. The coupled Schwinger-Dyson equations with two massive gauge bosons are specified by the eight parameters in the improved ladder approximation: gauge boson masses M_A and M_B, Yang-Mills interaction scales $\Lambda = \Lambda_A, \Lambda_B$ of the running couplings, one-loop β functions $\mu dg/d\mu = \beta(g) = -\beta_0 g^3$ for $g = g_A, g_B$, and the second Casimir invariants $C_2(F)$ of the fermion fundamental representations $F = \underline{N}_A, \underline{N}_B$. Here, we are using the Yang-Mills interaction scale for specifying the coupling strengths. When we solve the coupled Schwinger-Dyson equations, one dimensionful parameter out of M_A, M_B, Λ_A and Λ_B is irrelevant. We regard Λ_B as a unit scale during the calculation by rescaling all dimensionful parameters in terms of Λ_B. In the present system the coefficient of the β function and the second Casimir invariant have different values in the two Schwinger-Dyson equations. Namely, $\beta_{A0} = (11N_A - 2N_B)/(3\cdot 16\pi^2)$, $\beta_{B0} = (11N_B - 2N_A)/(3\cdot 16\pi^2)$ and $T^a_X T^a_X = C_2(\underline{N}_X) = (N_X^2 - 1)/(2N_X)$ for $X = A, B$.

Now, let us write down the coupled Schwinger-Dyson equations. The fermion propagator is defined by

$$S_F = \begin{pmatrix} \langle \psi_R \overline{\psi}_R \rangle & \langle \psi_R \overline{\xi}_L \rangle & \langle \psi_R \overline{\eta}_L \rangle \\ \langle \xi_L \overline{\psi}_R \rangle & \langle \xi_L \overline{\xi}_L \rangle & \langle \xi_L \overline{\eta}_L \rangle \\ \langle \eta_L \overline{\psi}_R \rangle & \langle \eta_L \overline{\xi}_L \rangle & \langle \eta_L \overline{\eta}_L \rangle \end{pmatrix} . \tag{3.6}$$

We note that the free fermion propagator is given by

$$S_{F0}(p) = \frac{i}{\alpha^\mu p_\mu} , \tag{3.7}$$

where we define generalized σ matrices as

$$\alpha^\mu = \begin{pmatrix} \overline{\sigma}^\mu & 0 & 0 \\ 0 & \sigma^\mu & 0 \\ 0 & 0 & \sigma^\mu \end{pmatrix} , \quad \overline{\alpha}^\mu = \begin{pmatrix} \sigma^\mu & 0 & 0 \\ 0 & \overline{\sigma}^\mu & 0 \\ 0 & 0 & \overline{\sigma}^\mu \end{pmatrix} , \tag{3.8}$$

with

$$\overline{\sigma}^\mu = \sigma_\mu = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) . \tag{3.9}$$

Then, the Schwinger-Dyson equation is given by

$$iS_{F0}^{-1}(p) - iS_F(p)^{-1} = \int \frac{d^4k}{(2\pi)^4 i}\, g_A^2(-p^2 - k^2)\, \alpha^\mu T^a_A\, iS_F(k)\, \alpha^\nu T^a_A\, K_{\mu\nu}(p-k; M_A)$$

$$+ \int \frac{d^4k}{(2\pi)^4 i}\, g_B^2(-p^2-k^2)\, \alpha^\mu T_B^a\, iS_F(k)\, \alpha^\nu T_B^a\, K_{\mu\nu}(p-k;M_B) \ . \tag{3.10}$$

The generators T_A^a and T_B^a eliminate $SU(N_A)_A$ and $SU(N_B)_B$ singlet states, respectively. The propagator $K_{\mu\nu}(l;M)$ of a massive gauge boson in a Landau-like gauge[18] is given by

$$K_{\mu\nu}(l;M) = \frac{1}{M^2-l^2}\left(g_{\mu\nu} + \frac{l_\mu l_\nu}{M^2-l^2}\right) \ . \tag{3.11}$$

The quantity $g_X^2(\mu^2)$ $(X = A, B)$ is the running coupling having a threshold scale at the gauge boson mass $\mu = M_X$. There is no need to regularize the running coupling g_X^2 below the scale Λ_X if $M_X > \Lambda_X$, since the running of g_X^2 stops below the scale M_X. Then, g_X^2 takes the form

$$g_X^2(\mu^2) = \frac{1}{\beta_{X0}\ln(\max(\mu^2, M_X^2)/\Lambda_X^2)} \ , \tag{3.12}$$

when we work in $M_X > \Lambda_X$. In the cases with a massless gauge boson the running coupling should be regularized, and various forms[19, 20, 21] may be taken. When we work in $M_X \le \Lambda_X$, we adopt the following form[20, 4]

$$g_X^2(\mu^2) = \frac{1}{\beta_{X0}} \times \begin{cases} \dfrac{1}{t} & \text{if } t_F < t \\ \dfrac{1}{t_F} + \dfrac{(t_F-t_C)^2-(t-t_C)^2}{2t_F^2(t_F-t_C)} & \text{if } t_C < t < t_F \\ \dfrac{1}{t_F} + \dfrac{(t_F-t_C)}{2t_F^2} & \text{if } t < t_C \end{cases} \ , \tag{3.13}$$

where $X = A, B$, $t = \ln(\max(\mu^2, M_X^2)/\Lambda_X^2)$ and we fix $t_F = 0.15$ and $t_C = -2.0$.

Next, let us transform the SD equation in a component form. We have vanishing condensates between the Weyl fermions with the same chiralities:

$$\langle\overline{\psi}_R\psi_R\rangle = \langle\overline{\xi}_L\xi_L\rangle = \langle\overline{\eta}_L\eta_L\rangle = \langle\overline{\xi}_L\eta_L\rangle = \langle\overline{\eta}_L\xi_L\rangle \equiv 0 \ . \tag{3.14}$$

Then, the non-trivial condensates are $\langle\overline{\psi}_R\xi_L\rangle = \langle\overline{\xi}_L\psi_R\rangle$ and $\langle\overline{\psi}_R\eta_L\rangle = \langle\overline{\eta}_L\psi_R\rangle$, where the condensates are taken as real numbers by using phase transformations of the fermion fields. Then, the mass function takes the form

$$\Sigma = \begin{pmatrix} 0 & 1_{B+\xi}\Sigma_A & 1_{A+\eta}\Sigma_B \\ 1_{B+\xi}\Sigma_A & 0 & 0 \\ 1_{A+\eta}\Sigma_B & 0 & 0 \end{pmatrix} \ , \tag{3.15}$$

where $1_{B+\xi}$ and $1_{A+\eta}$ are $N_B \times N_B$ and $N_A \times N_A$ unit matrices, respectively. In the Landau-like gauge the wave function renromalizations are expected to be small, then the fermion propagator is given by

$$S_F(p) = \frac{i}{\alpha^\mu p_\mu - \Sigma(-p^2)} = (\overline{\alpha}^\mu p_\mu + \Sigma(-p^2))\frac{-i}{\Sigma(-p^2) - p^2} \ . \tag{3.16}$$

Substituting Eq. (3.16) into Eq. (3.10) and carrying out the four-dimensional angle integrations, we find the coupled Schwinger-Dyson equations in component form

$$\begin{aligned} \Sigma_A(x) &= \int_0^\infty y dy K_A(x,y)\frac{\Sigma_A(y)}{y + \Sigma_A(y)^2 + \Sigma_B(y)^2} \ , \\ \Sigma_B(x) &= \int_0^\infty y dy K_B(x,y)\frac{\Sigma_B(y)}{y + \Sigma_A(y)^2 + \Sigma_B(y)^2} \ , \end{aligned} \tag{3.17}$$

where the kernel K_X $(X = A, B)$ is given by

$$K_X(x,y) = \frac{\lambda_X(x+y)}{x + y + M_X^2 + \sqrt{(x+y+M_X^2)^2 - 4xy}}\left(3 + \frac{M_X^2}{\sqrt{(x+y+M_X^2)^2 - 4xy}}\right) \ , \tag{3.18}$$

with

$$\lambda_X(x) \equiv \frac{C_2(\underline{N}_X)g_X^2(x)}{8\pi^2} \ . \tag{3.19}$$

4 Nambu-Goldstone Boson and its Decay Constant

In this section we briefly show how the NG boson couples to the gauge current with the decay constant and we derive a generalized Pagels-Stokar formula in the present system. In this paper instead of solving the BS equation for the NG boson π_A we use a convenient approximation by Pagels and Stokar[22], in which the BS amplitude is entirely given by the mass function. If we omit the interference effect of the other mass function, our formula reduces to the usual Pagels-Stokar formula[22] up to an overall factor.

In order for the argument to be transparent we concentrate on the dynamical symmetry breaking of the gauge group $SU(N_B)_B$. The Noether current of $SU(N_B)_B$ is given by

$$J_B^{b\mu} = \overline{\psi}_R \overline{\sigma}^\mu T_B^b \psi_R + \overline{\eta}_L \sigma^\mu T_B^b \eta_L \ , \tag{4.20}$$

where T_B^b is the generator of $SU(N_B)_B$. The NG boson π_A couples to the first part of this current (4.20), and also couples to the left Weyl spinor current, as

$$\begin{aligned} \langle 0|\overline{\psi}_R \overline{\sigma}^\mu T_B^b \psi_R|\pi_A^a(q)\rangle &= i\delta^{ab} q^\mu \ f_A \\ \langle 0|\overline{\xi}_L \sigma^\mu T_\xi^b \xi_L|\pi_A^a(q)\rangle &= -i\delta^{ab} q^\mu \ f_A \ . \end{aligned} \tag{4.21}$$

where f_A is its decay constant.

The BS amplitude of the NG boson π_A is defined by

$$\langle 0|T\Psi_i(x/2)\overline{\Psi}_j(-x/2)|\pi^a_A(q)\rangle \equiv 1_A\, T^a_{B+\xi} \int dx e^{-ipx}\chi_{Aij}(p;q)\ , \tag{4.22}$$

where $\Psi = (\psi_R, \xi_L, \eta_L)$ and 1_A is the $N_A \times N_A$ unit matrix and denotes $SU(N_A)_A$ gauge singlet. The truncated BS amplitude $\widehat{\chi}^a_A(p;q)$ is defined by

$$\widehat{\chi}^a_A(p;q) = S_F^{-1}(p+q/2)\,\chi^a_A(p;q)\,S_F^{-1}(p-q/2)\ . \tag{4.23}$$

Then, from Eqs. (4.21) and (4.22) the decay constant f_A is expressed in terms of the truncated BS amplitude $\widehat{\chi}_A$ as

$$q^\mu\ f_A = \frac{N}{2}\int\frac{d^4p}{(2\pi)^4 i}\ \mathrm{tr}\ \overline{\sigma}^\mu\Big[iS_F(p+q/2)\ \widehat{\chi}_A(p;q)\ iS_F(p-q/2)\Big]_{11}\ . \tag{4.24}$$

The chiral Ward-Takahashi identity for the "external" symmetry $SU(N_B)_B \times SU(N_B)_\xi$ is given by

$$-\,iq^\mu\Gamma^a_{A\mu}(p;q) = T^a_{B+\xi}\left(S_F^{-1}(p+q/2)\gamma_5' - \gamma_5' S_F^{-1}(p-q/2)\right)\ , \tag{4.25}$$

where $\gamma_5' = \mathrm{diag.}(1,-1,0)$. The NG boson π^a_A couples to this vertex function as

$$\Gamma^a_{A\mu}(p;q) \sim -2f_A\, T^a_{B+\xi}\ \widehat{\chi}(p;q)\ \frac{q_\mu}{q^2} + \cdots\ . \tag{4.26}$$

Using Eqs. (4.25) and (4.26), the truncated BS amplitude is given in terms of the mass function in the soft momentum limit $q_\mu \to 0$:

$$T^a_{B+\xi}\widehat{\chi}^a_A(p;0) = \frac{T^a_{B+\xi}}{f_A}\begin{pmatrix} 0 & -\Sigma_A(-p^2) & 0 \\ \Sigma_A(-p^2) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ . \tag{4.27}$$

In the Pagels-Stokar approximation we use the amputated BS amplitude in the soft momentum limit instead of the full one. Substituting Eq. (4.27) into Eq. (4.24) and expanding the propagators in terms of q_μ, we finally find

$$f_A^2 = \frac{N_A}{16\pi^2}\int_0^\infty xdx\ \Sigma_A(x) \times \frac{\Sigma_A(x)\left(1+\frac{4x\Sigma_B'(x)\Sigma_B(x)-\Sigma_B(x)^2}{8x}\right) - \frac{x}{2}\Sigma_A'(x)\left(1+\frac{\Sigma_B(x)^2}{2x}\right)}{x+\Sigma_A(x)^2+\Sigma_B(x)^2}\ . \tag{4.28}$$

The above arguments similarly hold for the dynamical breaking of the gauge symmetry $SU(N_A)_A$, and we obtain a similar formula for f_B.

5 Broken Dynamics in the Strong Coupling Phase and the large anomalous dimension γ_m

In section 2 we find phase diagrams ($N_A \leq 9$) in which a broken dynamics cannot break the other gauge symmetry. In this section we study whether the broken dynamics is in a strong coupling phase enough to break the $SU(2)_L$ symmetry. For definiteness we consider the case where the symmetry $SU(N_A)_A$ is manifest and the symmetry $SU(N_B)_B$ is broken dynamically, and we put $N_B = 3$. The topcolor gauge symmetry may be this $SU(3)_B$. We consider the case when the $SU(2)_L$ gauge interaction is relatively weak enough for us to regard it as a global symmetry.

Let us consider three Weyl fermions which are $SU(N_A)_A$ singlet. The fermions have the following charge:

$$\begin{array}{c|cc} & SU(3)_B & SU(2)_L \\ \hline q_L & 3 & 2 \\ t_R & 3 & 1 \\ b_R & 3 & 1 \end{array} \tag{5.29}$$

The following analysis is devoted to make clear whether the condensates $\langle \overline{t}_R q_L \rangle = \langle \overline{b}_R q_L \rangle$ form and break the $SU(2)_L$ symmetry.

The Schwinger-Dyson equation is simple and determines the propagator of the fermions q_L and q_R. We use the notations $q_L^i \equiv (t_L, b_L)^T$ and $q_R^i \equiv (t_R, b_R)^T$. We use the improved ladder approximation and the Landau-like gauge[18]. The Dirac fermion q respects the $SU(2)_{L+R}$ custodial symmetry, and the propagator takes the form

$$\langle q^i(x) \overline{q}_j(0) \rangle \equiv \delta^i_j S_F(x) \ . \tag{5.30}$$

The Dirac fermion propagator is expanded into two invariant amplitudes as

$$i S_F^{-1}(p) = A(-p^2) \not{p} - B(-p^2) \ . \tag{5.31}$$

The Schwinger-Dyson equation determines the invariant amplitudes $A(x)$ and $B(x)$, where $x \equiv -p^2$.

If we work with the massless gauge boson $M_B = 0$, the amplitudes $A(x)$ is identical to unity, which is shown after the four dimensional angle integration. Even when we work with $M_B \neq 0$, it is verified in Ref. [23] that $A(x) \simeq 1$ by an explicit numerical calculation in the fixed coupling case. In the high energy region $A(x)$ must converge to unity quickly enough, otherwise the resultant Dirac fermion propagator will be inconsistent with the

result by the operator product expansion and the renormalization group analysis. It means an explicit breaking of the chiral gauge symmetries in the present system.[19, 24] In this paper, we put $A(x) = 1$ for simplicity although the coupling is running. It should not modify the physical consequences of this paper.

Then, the Schwinger-Dyson equation takes the form

$$\Sigma(x) = \int_0^\infty xdx\ K_B(x,y) \frac{\Sigma(y)}{y + \Sigma(y)^2}\ . \tag{5.32}$$

After obtaining the fermion propagator $S_F(p)$, we estimate the decay constant f_t of the NG boson π_t^a by using the Pagels-Stokar formula[22] which is given by

$$f_t^2 = \frac{3}{16\pi^2} \int_0^\infty xdx\ \Sigma(x) \frac{\Sigma(x) - x\Sigma'(x)/2}{x + \Sigma(x)^2}\ . \tag{5.33}$$

We regard the decay constant as a function of M_B; i.e., $f_t = f_t(M_B)$. We show the decay constant $f_t(M_B)$ in Fig. 9. The decay constant is evaluated with $M_B > \Lambda_B$, where we

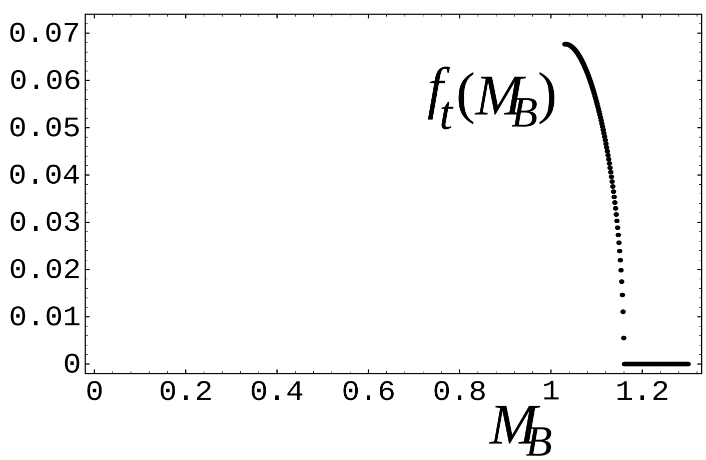

Figure 9: The decay constant f_t as a function of the gauge boson mass M_B. Unit is Λ_B. In the case $M_B > \Lambda_B$, the decay constant can be calculated without regularizing the infrared form of the running coupling. The phase transition occurs at $M_B = 1.16\Lambda_B$ with second order.

put $\Lambda_B = 1$. This result has no ambiguity stemming from any regularized form of the running coupling in the infrared region lower than the interaction scale Λ_B. This is in contrast with the result in Ref. [20]. Above the critical mass $M_B > f_t^{-1}(0) = 1.16\Lambda_B$ the chiral symmetry is restored. Near the phase transition point $(M < f_t^{-1}(0))$ with a small dynamical mass of the Dirac fermion, the anomalous dimension γ_m approaches that of the Nambu-Jona-Lasinio model:

$$\gamma_m \equiv -\frac{d}{d\ln\mu} \ln|\langle\overline{q}q\rangle| \cong \frac{d}{d\ln M_B} \ln|\langle\overline{q}q\rangle| = 2\ , \tag{5.34}$$

in the low energy region $\mu << M_B$. Here the gauge boson mass M_B plays the role of the cutoff. We conclude that *the renormalizable theory, considered here, provides the large anomalous dimension $\gamma_m \cong 2$ just below the critical point $M_B \cong f_t^{-1}(0)$ in the low energy region.*

Next, let us proceed to the lower region $M_B < \Lambda_B$. We can guess that $f_t(M_B)$ will be a constant in $M_B < \Lambda_B$, since such a mass M_B smaller than Λ_B should be negligible in the gauge interaction dynamics. This is also confirmed empirically. If we convert the value $f_t(\Lambda_B)/\Lambda_B$ to the ordinary QCD case by multiplying by a necessary factor, it already saturates the experimental value 93 [MeV]/Λ_{QCD} of the actual pion. Therefore, we should keep the property $f_t(M_B) \sim$ const. ($M_B < \Lambda_B$) when we regularize the running coupling in the infrared region. We adopt the form (3.13). Of course we should use the

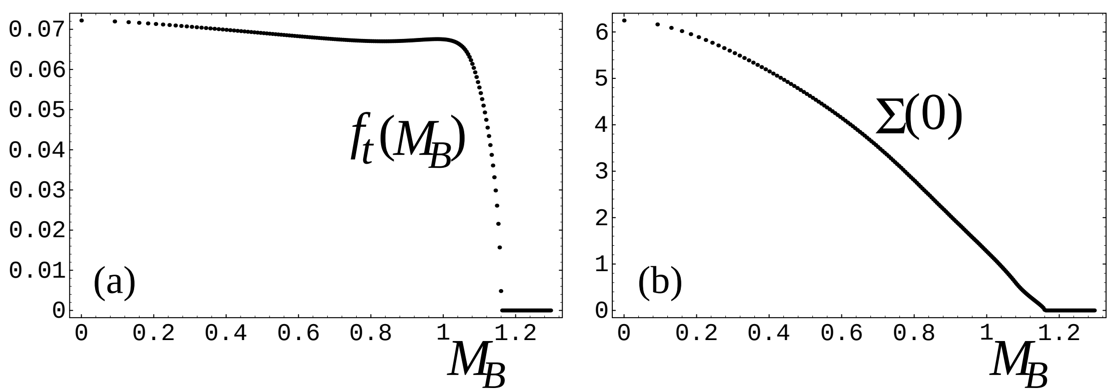

Figure 10: The decay constant f_t is plotted as a function of the gauge boson mass M_B in Fig. (a). The mass function of the top quark is plotted in Fig. (b). The unit is in Λ_B. As mentioned in Fig. 9 the phase transition occurs at $M_B = 1.16\Lambda_B$.

same coupling as that used in Eq. (3.10). Above the scale $\mu^2 = \exp t_F$ the functional form is exactly the same as that of the one-loop running coupling, below the scale $\mu^2 = \exp t_F$ the running coupling is regularized by using the second order polynomial in $t = \ln \mu^2$ and in the low energy region $t \leq t_C$ the coupling becomes a constant. The running coupling with $t_F = 0.15$ agrees with the one-loop running coupling form over almost of the range of t greater than the interaction scale Λ_B. The smaller value of t_F would be good but increases the error in numerical calculations. The result of $f_t(M_B)$ is shown in Fig. 10. We should notice that the functional form above $M_B^2 \geq \exp t_F \simeq 1.16$ in Fig. 10 is exactly the same as that in Fig. 9, and is perfectly independent on the infrared regularization

of the running coupling, since the running of the coupling stops below the threshold M_B^2 which is just above the regularized scale $\exp t_F$. We observe that $f_t(0) = 0.0722$ and $f_t(\Lambda_B) = 0.0675$. So, the decay constant squared f_t does not change by more than 7% in the range $0 < M_B < \Lambda_B$.

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